striction on the computer sharing such tasks would be the total computational speed requirements.

The checklist management function was added to the system to demonstrate the integration potential of the stall margin indicator.

Conclusion

The stall margin indicator described in this paper was developed as a flexible test bed. The hardware has been designed to provide the software with as much control over the system as possible, thus allowing new innovations to be implemented with minimal reconfiguration of the hardware. The system will be used for continued research into effective application of stall margin indication as a useful pilot information source. The display technology and configuration will be studied along with effective integration of stall margin with other cockpit functions.

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References

[1]Hoadley, A., "Conversion of Wing Surface Pressure Into Normalized Lift Coefficient," SAE 790567, April 1979; also, SAE Transactions, 1979.

[2]Hoadley, A., "Stall Margin Indicator Development," AIAA Paper 86-2694, Oct. 1986.

[3]Hoadley, A., "Normalized Coefficient of Lift Indicator," U.S. Patent No. 4,325,104.

[4]Hoadley, A., "Wing Mounted Stall Condition Detector," U.S. Patent No. 4,350,314.

Evaluation of Two Singular Integrals fromThin Airfoil Theory

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Introduction

In the analysis of thin airfoils using second-order small-perturbation potential flow theory, a need arises to evaluate integrals of the type

$$\int_{-1}^{1} \frac{\xi^n \ln\sqrt{(1+\xi)/(1-\xi)}}{\sqrt{1-\xi^2}\,(x-\xi)}\,d\xi \tag{1}$$

and

$$\int_{-1}^{1} \frac{\xi^n \ln\sqrt{(1+\xi)/(1-\xi)}}{(x-\xi)}\,d\xi \tag{2}$$

for $-1 < x < 1$.

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These integrals were evaluated in closed form by Van Dyke and are available in Appendix B of Ref. 1.

However, in airfoil theory one frequently works in terms of the Glauert variables θ, ϕ, defined as

$$\theta = \cos^{-1}x, \qquad \phi = \cos^{-1}\xi \tag{3}$$

In these variables, integrals (1) and (2) may be expressed, respectively, in terms of sums of integrals of the type

$$H_n(\theta) \equiv \int_0^{\pi} \frac{\cos n\phi \ln \cot\phi/2}{\cos\phi - \cos\theta}\,d\phi \tag{4}$$

and

$$I_n(\theta) \equiv \int_0^{\pi} \frac{\sin n\phi \ln \cot\phi/2}{\cos\phi - \cos\theta}\,d\phi \tag{5}$$

where $0 < \theta < \pi$.

In this brief Note, these two integrals are evaluated in closed form.

Evaluation of the Integrals

Integral H_n

For this integral, the following recurrence relation ($n \geq 0$ and integer) can be derived:

$$H_{n+1}(\theta) + H_{n-1}(\theta) = 2\cos\theta H_n(\theta) + 2\int_0^{\pi} \cos n\phi \ln \cot\frac{\phi}{2}\,d\phi \tag{6}$$

Now,

$$\int_0^{\pi} \cos n\phi \ln \cot\frac{\phi}{2}\,d\phi = \int_0^{\pi} \frac{\sin n\phi}{n\sin\phi}\,d\phi = \begin{cases} 0 & \text{if } n \text{ is even} \\ \pi/n & \text{if } n \text{ is odd} \end{cases} \tag{7}$$

Hence, Eq. (6) may be rewritten as

$$H_{n+1}(\theta) + H_{n-1}(\theta) = 2\cos\theta H_n(\theta) + \left[1-(-1)^n\right]\pi/n \tag{8}$$

whose solution is

$$H_n(\theta) = A\sin n\theta + B\cos n\theta + \pi\sum_{k=1}^{n-1} \frac{\left[1-(-1)^k\right]}{k}\,\frac{\sin(n-k)\theta}{\sin\theta} \tag{9}$$

The two conditions needed to evaluate the constants A and B are obtained by evaluating H_0 and H_1, which are

$$H_0(\theta) = \pi^2/(2\sin\theta) \tag{10}$$

$$H_1(\theta) = \pi^2\cot\theta/2 \tag{11}$$

from which

$$A = 0, \qquad B = \pi^2/(2\sin\theta)$$

Therefore,

$$H_n(\theta) = \frac{\pi^2}{2}\,\frac{\cos n\theta}{\sin\theta} + \pi\sum_{k=1}^{n-1} \frac{\left[1-(-1)^k\right]}{k\sin\theta}\sin(n-k)\theta \tag{12}$$

Integral I_n

For this integral, the following recurrence relation ($n \geq 0$ and integer) holds:

$$I_{n+1}(\theta) + I_{n-1}(\theta) = 2\cos\theta I_n(\theta) + 2\int_0^{\pi} \sin n\phi \, \ell n \cot\frac{\phi}{2} \, d\phi \tag{13}$$

Now

$$\int_0^{\pi} \sin n\phi \, \ell n \cot\frac{\phi}{2} \, d\phi = \begin{cases} 0 & \text{if } n \text{ is odd} \\ \sum_{k=1}^{n/2} \frac{4}{n(2k-1)} & \text{if } n \text{ is even} \end{cases} \tag{14}$$

Hence, Eq. (13) may be rewritten as

$$I_{n+1}(\theta) + I_{n-1}(\theta) = 2\cos\theta I_n(\theta) + \frac{[1+(-1)^n]}{n} \sum_{k=1,3}^{n} \frac{4}{k} \tag{15}$$

whose solution is

$$I_n(\theta) = A \sin n\theta + B \cos n\theta + \sum_{k=1}^{n+1} \frac{[1+(-1)^k]}{k} \sum_{j=1,3}^{k} \frac{4}{j} \frac{\sin(n-k)\theta}{\sin\theta} \tag{16}$$

$I_0(\theta)$ and $I_1(\theta)$ can be directly calculated as

$$I_0(\theta) = 0 \tag{17}$$

$$I_1(\theta) = \pi^2/4 - \ell n^2 \cot\theta/2 \tag{18}$$

from which

$$A = \left(\frac{\pi^2}{4} - \ell n^2 \cot\frac{\theta}{2}\right)/\sin\theta, \qquad B = 0 \tag{19}$$

Therefore,

$$I_n(\theta) = \left(\frac{\pi^2}{4} - \ell n^2 \cot\frac{\theta}{2}\right) \sin n\theta/\sin\theta + \sum_{k=2,4}^{n-1} \sum_{j=1,3}^{k} \frac{8\sin(n-k)\theta}{jk\sin\theta} \tag{20}$$

Conclusion

Two Cauchy integrals useful in second-order thin airfoil theory have been evaluated in closed form.

Reference

[1]Van Dyke, M.D., "Second Order Subsonic Airfoil Theory Including Edge Effects," NACA R-1274, 1956.

Notice to Subscribers

We apologize that this issue was mailed to you late. As you may know, AIAA recently relocated its headquarters staff from New York, N.Y. to Washington, D.C., and this has caused some unavoidable disruption of staff operations. We will be able to make up some of the lost time each month and should be back to our normal schedule, with larger issues, in just a few months. In the meanwhile, we appreciate your patience.